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ESTIMATED COSTS OF EXTENDED LOW-RATE AIRFRAME PRODUCTION.(U)

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David J. Dreyfuss, Joseph P. Large

A Project AIR FORCE report
prepared for the
United States Air Force

Rand
SANTA MONICA, CA. 90406

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ABSTRACT

Achieving a high rate of production as quickly as possible has traditionally been viewed as the most effective way of satisfying time-urgent inventory requirements while keeping production costs low. One common consequence has been the delivery of less than fully qualified production articles. This report discusses the cost of extending initial low-rate production while tests of early production articles continue. The relatively small resultant cost increases can potentially be offset by the delivery of more capable production items, lessened needs for postdelivery modification or retrofit, and lower total-life system costs. (RP)

ABSTRACT

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Estimated Costs of Extended Low-Rate Airframe Production

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PREFACE

This report, prepared as part of the Project AIR FORCE Systems Acquisition Management Program, addresses the question of how a deliberately extended low initial production rate for systems or subsystems may affect program outcomes--cost, schedule, and operational performance.

Achieving a high rate of production as early as possible has traditionally been viewed by industry and the Air Force as the most effective way of satisfying time-urgent inventory needs while reducing the unit cost of production articles (and thus the total program cost). But an occasional result of this practice has been the delivery of marginally capable equipment, costly to maintain and deficient in performance. During an extended low-rate production phase, the results of tests and evaluations of early production items could be assessed and essential corrective changes incorporated in the production articles. Delivered articles would thereby more nearly satisfy Air Force needs. A principal uncertainty, addressed in this report, is the incremental cost of such an extension. The benefits arise in improved reliability, greater operational capability, and lessened maintenance needs.

The results of the study should be useful to the U.S. Air Force, other Department of Defense agencies, the National Aeronautics and Space Administration, and other organizations concerned with the processes of designing and producing technologically advanced systems and subsystems.

The work was done under the Project AIR FORCE study project "System Acquisition Policy Studies."

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SUMMARY

The delivery of operationally deficient equipment has been a persistent problem for the military services of the United States. The problem often arises in the inappropriate ordering and sequencing of test and production intervals. Early test results frequently indicate the need for modification of production specifications, but by the time the data have been digested and the need appreciated, production (and deliveries) may be well along.

One response to this set of problems would be to extend the period during which initial production continues at a relatively low rate, while concurrently testing early production articles and incorporating essential changes. This report examines the incremental cost of such extended low-rate production, using three rate variance models (developed by Rand, Fairchild Industries, and Northrop).

The total cost penalties associated with a *planned* phase of extended low-rate production do not appear to be excessive. They may add from 18 to 38 percent to the cost of the first 24 aircraft, for example. When viewed in the context of total system life cycle costs, such increased costs incurred in early phases of production could be offset by the benefits of later production at more efficient high rates and by the lessened post-delivery costs of modifying or retrofitting systems. In decisions involving adoption of a phased acquisition strategy that includes provision for extended low-rate early production, it does not appear that cost alone, and particularly the unit costs of early production systems, should be a dominant consideration.

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CONTENTS

I. INTRODUCTION

The delivery of operationally deficient equipment has been a persistent problem for the military services of the United States. That the problem often arises in the inappropriate ordering and sequencing of test and production intervals has long been acknowledged.[1] Early test results frequently indicate the need for modification of production specifications, but by the time the data have been digested and the need appreciated, production (and deliveries) may be well along. If deficiencies are uncovered after equipment has been accepted by operational units, the consequences can be highly disruptive. Incorporating modifications interrupts the production process, and items delivered early must frequently be modified, retrofitted, or operated under various constraints--or even discarded. Delaying deliveries until extended testing has been completed is viewed by operational forces as a distasteful alternative, even though establishing a high rate of initial production may lead to the delivery of "operational" equipment that is, in fact, nonoperational until modified, or even until solutions to latent performance, reliability, or maintenance problems can be found. And even when the delivered equipment is marginally capable, levels of performance sought by the operators may be unachievable.

One response to this set of problems would be to extend both the period during which production continues at a relatively low rate,

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[1] See, for example, *Report to the President and the Secretary of Defense on the Department of Defense by the Blue Ribbon Defense Panel*, Washington, D.C., July 1970; *Report of the Commission on Government Procurement* (Vol. II), Government Printing Office, Washington, D.C., December 1972; *Defense Science Board Task Force on R and D Management, Final Report on Systems Acquisition*, Washington, D.C., August 1969; A. Boykin, *Acquisition Strategy and Systems Management*, Aeronautical Systems Division, Wright-Patterson Air Force Base (unpublished); B. H. Klein et al., *The Role of Prototypes in Development*, The Rand Corporation, RM-3467/1-PR, April 1971; Robert Perry et al., *System Acquisition Strategies*, The Rand Corporation, R-733-PR/ARPA, June 1971; and Robert Perry, *A Prototype Strategy for Aircraft Development*, The Rand Corporation, RM-5597-1-PR, July 1972.

and the phase during which early production articles are extensively tested, with essential changes being incorporated as their need is confirmed.[2] Extended low-rate production would also permit field units to receive and exercise early production articles under operational conditions, thereby encouraging "lead the fleet" accumulation of experience on reliability, maintainability, and durability factors.

One conventional objection to such an approach has been that low-rate production is so costly that it negates any subsequent benefits from the early incorporation of changes suggested by test outcomes. Whether that is a valid objection, and to what extent, has not been widely considered. A comprehensive determination of costs and benefits would require surveys of a broad range of system and subsystem cost factors extending over the lives of many different articles. But it is conceivable that a less exhaustive exploration of cost-benefit factors could cast light on the potential of extended low-initial-rate production as a useful acquisition option for selective application. This report explores that possibility.

The approach used here is to estimate the real costs of low-rate production--of producing equipment, aircraft in this instance, for extended periods at rates much lower than is normally contemplated--and to compare those costs with the costs of "normal" programs. The cost-benefit estimation problem has been approached from two directions: first, by way of a Rand-developed statistical cost model that permits the separate estimation of each functional cost element in aircraft production, using production rate or the duration of production as an explanatory variable; second, by using production cost-estimating models developed by aircraft manufacturers. Two such models were available, both based on the rate variance formulas submitted to the Air Force by Northrop and Fairchild in the course of the competition between the A-9 and the A-10. Those formulas indicate how the two contractors perceived the effects of different production rates on

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[2] The notion is not new. It was initially proposed (as the "Cook-Craigie Plan") as a solution to problems encountered during development and production of the first generation of jet aircraft and was, in reality, the source of the "fly before you buy" expression later mistakenly applied to prototyping.

the first two proposed production lots of the aircraft. Each model is described and applied to different production options. The effect of production rate changes on each major cost element is shown for both models. Where direct comparisons were feasible, agreement was generally good. Agreement between the industry estimates and those derived from the Rand model was also good (for both individual cost elements and total costs). Such outcomes reinforce confidence that the magnitude of the cost penalties associated with low production rates can be estimated with reasonable accuracy.

But one important qualifier must be stated. Production rates as low as those postulated here were not contemplated when the rate variance formulas were derived, and the historical data base contains few examples of *planned* production at such low rates. When an unplanned low rate is imposed on a program by external events or circumstances, timely and appropriate adjustments of the actual workload may not occur and incurred costs are almost certain to be higher than would be the case if such a rate had been planned.

Both the Rand statistical model and the Northrop formula indicate that the cost of producing 24 aircraft over a period of 36 months is 18 percent greater than the cost of producing the same aircraft in 24 months. The Fairchild formula indicates that the cost penalty would be 38 percent. Are such upper-bound cost penalties greater than the costs generated by a premature commitment to high-rate production? The answer determines the value of a low-initial-rate production strategy. If common sense suggests that low-initial-rate production is best suited to programs that obviously involve technology of uncertain reliability, history responds that uncertainty of cost, schedule, and article performance outcomes is characteristic of *many* major system development-production programs, and that *most* major systems are initially delivered with varying degrees of debilitating performance limitations.

This study has been restricted to the investigation of imposed low production rates on airframes and the resulting cost consequences. Clearly, the benefits of a low airframe production rate could be lost if an inappropriate production rate for engines and avionics caused

the cost of the entire aircraft to increase. But there is no reason to believe that extended testing of engines and avionics would be non-beneficial, or that potentially higher unit costs of early production items would not be offset by lower costs of maintenance, by lesser needs for modification, and by other operational advantages inherent in higher quality operational subsystems. The advantages of a phased acquisition strategy that included a period of deliberately extended low-rate production could not be realized unless *all* elements of the system or subsystem being so developed and produced were subjected to the same careful planning and management.

II. THE ELEMENTS OF COST

Aircraft production cost accounting ordinarily accommodates eight elements: engineering, tooling, manufacturing labor and materials, quality control, purchased equipment, overhead, general and administrative cost (G&A), and fee or profit. Changes in production rate will influence the individual cost elements in different ways because some are fixed, others are variable, and some change from one state to the other as a program proceeds. *Fixed costs* are typically incurred at the beginning of a program; they do not vary as a result of quantity- or rate-change decisions made later. The costs of plant construction, equipment purchases, taxes, insurance, utilities, etc., are treated as fixed. *Semi-fixed* or *quasi-fixed* costs are stable for brief intervals but not over longer periods. They generally extend to the pay of white-collar workers and workers on contracts, the costs of tools that eventually wear out, and other commitments that cannot be readily adjusted to meet changing conditions. Overhead and general and administrative costs are also in that category. *Marginal costs* are incurred for resources purchased to satisfy short-time demands; they vary directly with output per unit of time. They typically include factory labor and most materials. Rising marginal costs are generated by such factors as plant congestion, delay of maintenance (and subsequent production halts), employing lower quality labor to increase production rates, and third-shift premiums. Falling marginal costs may stem from quantity discounts on materials and the adoption of more capital-intensive manufacturing processes. This last category of costs can also be called "true rate effects."

Average unit costs in a specified period are affected by production rate in either (or both) of two ways: true rate effects, and allocation of semi-fixed costs to production in a particular period. That is, the greater the quantity produced during any period, the lower will be the prorated semi-fixed costs per unit. *Most of the influence of production rate on aircraft costs is from semi-fixed*

costs.[1] This finding carries implications for interpreting the research results and for deriving policy conclusions. Programs that are both *planned* and *executed* at a given production rate--high or low--need not suffer severe cost penalties. But any deviation from planned rates can generate excess costs that will persist over some period of time. Examination of models and data must take these effects into account.

The major elements of cost are described below in conjunction with an explanation of how, in the context of the aircraft industry, they are likely to be affected by production rate changes.[2]

ENGINEERING DIRECT LABOR

Engineering hours are those expended in the study, analysis, design, development, evaluation, and redesign of an airframe. That activity includes the preparation of specifications, drawings, parts lists, and wiring diagrams; technical coordination between engineering and manufacturing; vendor coordination; test planning and scheduling; analysis of test results; and data reduction and report preparation. It also extends to the specification of requirements for reliability, maintainability, and quality control.

Such functions are relatively independent of quantity; they must be performed whether 20 airplanes are produced or 200. In a conventional program that proceeds according to plan, most engineering hours would be expended in the early stages. Figure 1 shows a typical progress curve. The cumulative total of engineering hours after seven

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[1] This is illustrated by the following linear model:

T = total costs; t = time; q = quantity; r = rate = q/t.
T = a + bq + ct + dr.

$$\left.\frac{\partial T}{\partial r}\right|_{q=\text{const}} = \frac{\partial T}{\partial T}\frac{\partial t}{\partial r} + \frac{\partial T}{\partial r} = \frac{-cq}{r^2} + d.$$

The cost effect of a change in rate (holding quantity constant) arises in two factors: costs that are constant for a brief time (semi-fixed costs) and pure rate effects.

[2] Cost-element definitions are based on those in the *Contractor Cost Data Reporting System*, Air Force Systems Command, Pamphlet 800-15, 5 November 1973.

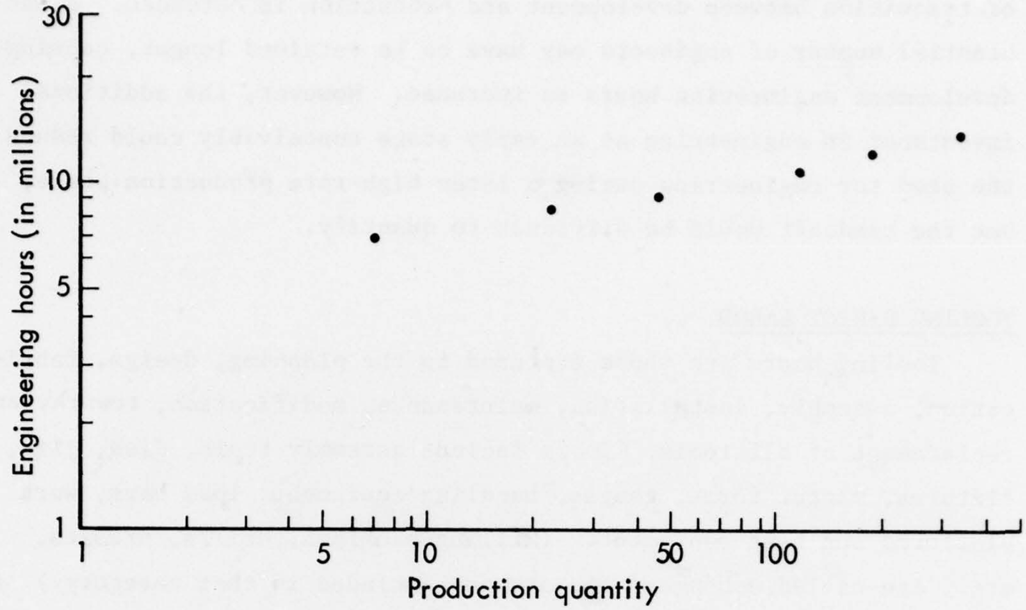

Fig. 1 — Cumulative engineering hours versus production quantity for a typical fighter aircraft

aircraft have been built is approximately half the total of hours after 340 aircraft have been built, which is consistent with expectations.

Several factors influence the total number of engineering hours expended in a program. Many engineering tasks are best performed in sequence, which means that any attempt to compress development time can lead to inefficiencies and a consequent increase in hours expended. Most projects provide some minimum number of engineers for each discipline (aerodynamics, stress analysis, thermodynamics, propulsion, etc.). Those engineers perform analyses, solve technical problems, do redesign, and seek means of product improvement. The number of engineers assigned to a program peaks during initial development stages, then usually begins to decline at the time of first flight. If the period

of transition between development and production is extended,[3] a substantial number of engineers may have to be retained longer, causing development engineering hours to increase. However, the additional investment in engineering at an early stage conceivably could reduce the need for engineering during a later high-rate production phase, but the tradeoff would be difficult to quantify.

TOOLING DIRECT LABOR

Tooling hours are those expended in the planning, design, fabrication, assembly, installation, maintenance, modification, rework, and replacement of all tools. *Tools* include assembly tools, dies, jigs, fixtures, master forms, gauges, handling equipment, load bars, work platforms and test equipment. (Milling machines, drills, presses, etc., are called *equipment* and are not included in that category.) An initial set of tools is fabricated to support manufacturing at some specified maximum rate of production (e.g., four aircraft per month). Recurring or sustaining tooling has a fixed component that is time-dependent and a variable component that is a function of quantity. The former will increase as program length increases; the latter is largely unaffected by production rate. On balance, we would not expect tooling costs to be affected greatly by keeping production rate low for an additional year or two.

MANUFACTURING DIRECT LABOR

Manufacturing or factory direct labor hours are those expended on or chargeable to such operations as production scheduling and expediting, fabrication, processing, subassembly, final assembly, reworking, modification, experimental production, and installation of parts and equipment (powerplants, boosters, electronic equipment explosives, and other ordnance items including government furnished

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[3] The phrase "development-production transition," as used hereafter in this report, is intended to identify the period during which the first lot of production-configuration aircraft is manufactured. The size of that lot may vary; the range considered here is between 13 and 48 aircraft.

equipment). Manufacturing hours are thus a function of production quantity; hours expended per aircraft decrease as quantity increases in accordance with the familiar learning-curve equation, $Y = aX^{-b}$, where Y = manufacturing hours per aircraft and X = quantity.

The equation makes no provision for rate effects, and hours should vary inversely with rate for several reasons. First, setup time is reduced. Setup is the preparation of machinery or tools to fabricate or assemble parts. Once a machine has been set up, it can produce or handle as many parts as needed. A high production rate means more parts per setup (because the number of parts run is usually the number needed over a specified period of time), and the cost of setup can be allocated to a larger number of units. Second, assembly and installation consists of a sequence of repetitive operations. As production rate increases and more workers are employed, each worker performs fewer operations. Theoretically, with fewer different tasks, the worker becomes more efficient as the number of different tasks to perform decreases, thus decreasing the number of labor hours per unit of production.

In the circumstances that prevail in the airframe industry, manufacturing labor hours and production rate are inversely related: As production rate increases, factory hours per pound of airframe decrease. There are exceptions to this rule, and the influence of rate is often overstated because it is interrelated with the effects of quantity. Typically, rate increases as production quantity increases until a peak rate is achieved, which leads to correlation in the data. Also, configuration changes occur frequently during aircraft production, and each causes a perturbation in the number of manufacturing man-hours. Thus production rate tends to decrease as each change is introduced.

MANUFACTURING MATERIALS

As the term is used here, manufacturing materials include raw and semifabricated material, purchased parts, and purchased equipment. Typical examples are: raw materials (sheets, bars, rods); semifabricated materials (wires, cables, fabrics, conduits, tubings); purchased

parts (fasteners, valves, hydraulic fittings, electrical fittings); and purchased equipment (motors, generators, batteries, landing gear, instruments).

Obviously, the cost of material relates directly to the number of aircraft procured, although, generally, material cost per unit decreases as quantity increases. In other words, the so-called learning effect applies to materials as well as labor. In part, this effect derives from savings in the cost of raw materials as spoilage and the amount of scrap material are reduced. More important, however, a cost reduction accompanies volume purchases of parts and equipment. Volume purchase is ordinarily possible only when the production rate is fairly high because of the reluctance of contractors to order materials in volume and stockpile them.[4] A reason sometimes given for associating material cost with rate is that the make-buy pattern changes with rate. At high production rates, some items that would otherwise be manufactured in-house are purchased, and specialized suppliers responding to relatively large orders are often able to produce those items more cheaply than can an aircraft manufacturer.

INDIRECT COST

Indirect cost, which includes the various categories of overhead and G&A, is a major part of the cost of developing and producing aircraft. Table 1 shows the supporting activities categorized as indirect costs in the *Contractor Cost Data Reporting System*. Some of those costs are fixed; others vary as a function of levels of employment. If a contractor has no other business, most or all fixed costs have to be charged to whatever system is being developed. (That possibility can be considered as an unlikely worst case because *some* recurring business is necessary if a company is to survive.) Variable indirect costs are a much larger component of overhead. Views about their degree of variability differ, but the data gathered here suggest that a change in direct cost is accompanied by almost immediate and similar change in indirect cost.

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[4]Uncertainty of demand and cash-flow considerations largely explain such reluctance.

Table 1

OVERHEAD COST CATEGORIES[a]

Indirect Labor
- Salaries/wages
- Supplemental allowances
- Apprentice and OJT
- Administration and supervision
- Other

Employee Benefits
- Paid absences
- Employee insurance
- Savings-retirement plans
- Education
- Other

Payroll Taxes
- FICA
- Federal and state unemployment
- Composite payroll taxes
- Other

Employment
- Employee advertising
- Recruitment travel
- Employee relocation
- Composite employment
- Other

Communication/Travel
- Telephone and telegraph
- Postage
- Travel
- Corporate aircraft
- Other

Production Related
- Expendable tools and equipment
- Freight
- Material handling
- Manufacturing supplies/services
- Product servicing
- Tool handling
- Medical services
- Other

Facilities--Building/Land
- Depreciation and amortization
- Rentals
- Maintenance
- Insurance
- Utilities
- Property taxes
- Plant rearrangement
- Plant security
- Other

Facilities--Furniture/Equipment
- Depreciation and amortization
- Rentals
- Maintenance
- Data processing services
- Other

Administration
- Office supplies
- Reproduction/engineering supplies
- Professional services
- Contributions
- Other taxes
- Dues, memberships, and subscriptions
- Conventions and meetings
- Office services
- Other

Future Business
- Bid and proposal
- Independent research and development
- Advertising
- Other promotions

Other Miscellaneous
- Assessment and transfers
- Employee awards
- Corporate allocations
- Patents and royalties
- Other

Credits
- Transfers to other divisions
- Cash discounts
- Other

[a]Taken from *Contractor Cost Data Reporting System*.

Figure 2 shows the relationship among fixed, indirect, and direct cost over a 12-year period for an aerospace concern referred to hereinafter as Company A. Fixed cost tends to increase over time, primarily because of inflation, but it is insensitive to wide fluctuations in direct cost. Over the time span shown, the fixed cost peaked at about $17 million, which in that year represented about 14 percent of the total. When total cost exceeded $160 million, fixed cost accounted for about 10 percent.

Fixed costs were not available for any other company in a comparable time-series, but fragmentary data suggest that fixed costs typically constitute a small proportion of overhead. The breakdown is illustrated on the following page.

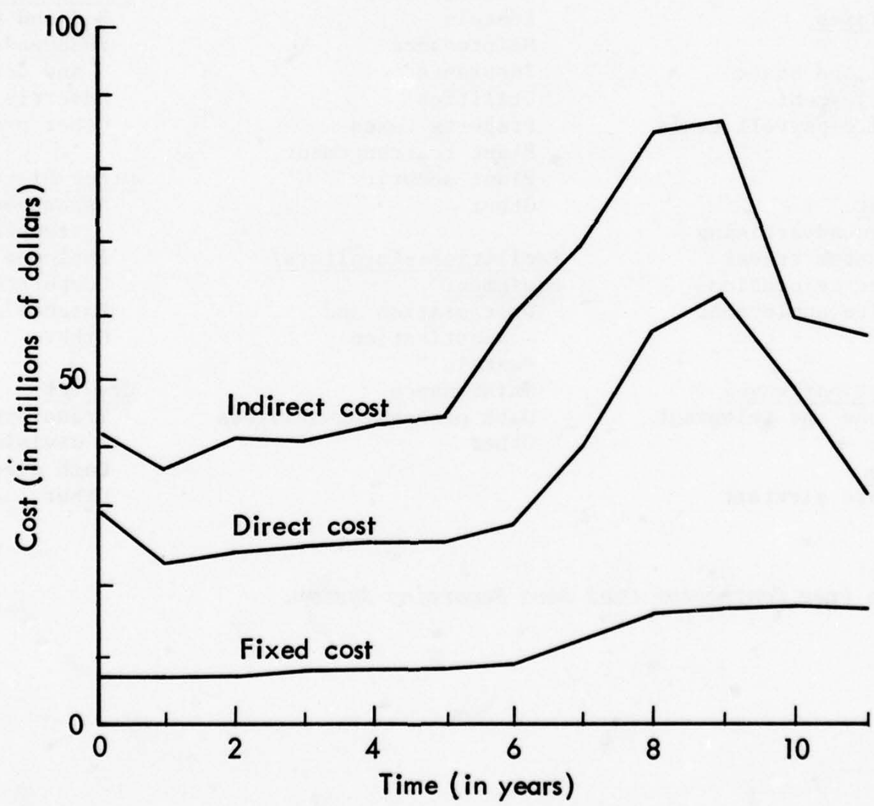

Fig. 2—Fixed, indirect, and direct cost versus time: Company A

Cost Element	Percent of Overhead
Fringe benefits	30
Indirect labor	35
Other controllables	15
Fixed	20
Total	100

Because fringe benefits are a function of both direct and indirect employment, it is sometimes argued that fringe costs incurred for direct labor should not be treated as overhead. But the airframe industry mostly continues to do so.

III. THE RAND MODEL

An earlier Rand study that examined the effect of production rate on the various elements of aircraft cost[1] concluded that engineering and overhead costs were more sensitive to rate changes than manufacturing costs. No attempt was made in that study or in a subsequent report on aircraft airframe cost-estimating procedures[2] to quantify the rate effect because the realized rate is rarely the rate used as an estimating variable in planning studies. Characteristically, anticipated production rates are not realized. One recent study showed that only slightly more than half as many military aircraft originally planned were actually manufactured during the first three years of production. On the average, initially planned peacetime production rates exceeded achieved rates by 59 percent.[3]

The present study examines the cost (in hours and dollars) of producing early lots of aircraft at different rates, using data not available for the previous study. We examined each functional cost element for a statistical sample of about 30 military aircraft programs and for several individual aircraft programs.

ENGINEERING HOURS

To determine the sensitivity of engineering hours to rate of output, cumulative total engineering hours for a sample of 24 U.S. military aircraft (ranging in airframe unit weight from 5000 to 279,000 lb and in first flight date from 1946 to 1972) were used to interpolate values for each aircraft model at 25, 50, 100, and 200 units. Engineering hours for units 1-25, 26-50, 51-100, and 101-200 were used as

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[1] J. P. Large et al., *Production Rate and Production Cost*, The Rand Corporation, R-1609-PA&E, December 1974.

[2] J. P. Large et al., *Parametric Equations for Estimating Aircraft Airframe Costs*, The Rand Corporation, R-1693-1-PA&E, February 1976.

[3] *Acceptance Rates and Tooling Capacity for Selected Military Aircraft*, Office of Assistant Secretary of Defense (Program Analysis and Evaluation), Washington, D.C., October 1974.

dependent variables in a series of regression analyses that included weight, speed, and rate as independent variables. Monthly production rate was not found to be a useful parameter because it changed from month to month, but delivery period (the number of months from delivery of the first aircraft in a lot to delivery of the last) was useful because it averaged out the short-term variations. As shown by Table 2, however, delivery period became less significant after 50 aircraft had been completed.[4]

It seems clear that for the development-production transition that typically occurs before 50 aircraft have been produced, delivery period has been a factor in the number of hours expended, but the regression equation may have little predictive value. First, the equation implies that engineering hours can always be reduced by shortening the acceptance period (e.g., total hours for a 12-month period would be only 68 percent of those for a 24-month period). Selecting the length of an acceptance period also requires accommodating to the realities of the program, and for modern high-performance military aircraft one would expect engineering hours to increase rather than decrease as the acceptance period decreases below some reasonable number. Second, while the evidence may be persuasive that engineering hours and program length go hand in hand, the relationship may arise in the fact that difficult programs take longer and require expenditure of more engineering hours per unit of time than do simple programs. Therefore, an increase in hours attributed to an increase in program length could also reflect an increase in program difficulty. Third, engineering time needs could have been estimated against a schedule different from that actually used. Had the original plan called for a *longer* schedule, fewer engineers might have been required and fewer engineering hours expended. Some lengthy programs reported below-average engineering hours (the A-3, F-105, and C-133 are examples; the regression equation overstates engineering hours for those aircraft). Thus, while the equation is a useful expression of what has happened in the past, it may not be a useful guide to the future.

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[4]The appendix contains a full statement of the statistical parameters for all equations in this section.

Table 2

ENGINEERING HOURS AS A FUNCTION OF AIRFRAME
UNIT WEIGHT, SPEED, AND DELIVERY PERIOD
($\text{Ln } Y = \text{Ln } A + B \text{ Ln } X1 + C \text{ Ln } X2 + D \text{ Ln } X3$)

Aircraft Production Quantities	Constant	Weight	Speed	Delivery Period	
Aircraft 1-25					
Value	.001	.713	1.051	.440	
T-ratio	-3.897	8.992	5.577	2.230	
Significance level	.001	.000	.000	.037	
R^2					.826
Standard error (%)					+42, -30
Aircraft 26-50					
Value	.002	.586	.911	.538	
T-ratio	-2.886	5.548	3.881	3.508	
Significance level	.009	.000	.001	.002	
R^2					.792
Standard error (%)					+53, -35
Aircraft 51-100					
Value	.000	.722	1.265	.192	
T-ratio	-4.052	6.551	4.767	1.111	
Significance level	.001	.000	.000	.281	
R^2					.800
Standard error (%)					+54, -35
Aircraft 101-200					
Value	.001	.688	.952	.152	
T-ratio	-2.632	5.239	3.258	.728	
Significance level	.020	.000	.006	.479	
R^2					.738
Standard error (%)					+54, -35

TOOLING

Because an initial set of tools would serve the range of production rates and quantities considered here, nonrecurring tooling hours would be the same in all cases. Sustaining tooling hours (for tool maintenance, modification, rework, and replacement) are partially a function of program length and partially a function of the number of parts manufactured. A regression analysis of tooling hours similar to that described for engineering hours, using the acceptance period for units 1-25 as an independent variable, produced the results displayed in Table 3.

Delivery period as a proxy for production rate is not statistically significant, and the other statistical properties are unimpressive. This is not to say that production rate has no effect on total tooling hours. The problem is that rate can affect tooling hours differently for different programs depending on how rate is planned and how it is achieved. However, even when rate effects are examined in a single program, we did not find them to be statistically significant. Consequently, despite the statement above that tooling hours have a time-dependent component, in this study they are treated as a function of production quantity only.

Table 3

TOOLING HOURS FOR AIRCRAFT 1-25 AS A FUNCTION
OF AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD
(Ln Y + Ln A + B Ln X1 + C Ln X2 + D Ln X3)

Statistical Values	Constant	Weight	Speed	Delivery Period	
Value	.063	.692	.527	.253	
T-ratio	-1.309	6.671	2.178	1.269	
Significance level	.202	0.000	0.039	0.216	
R^2					.670
Standard error (%)					+62, -60

MANUFACTURING LABOR

Most of the discussion about the effect of production rate on manufacturing hours focuses on quantities and rates much higher than those considered here. When the quantity involved is only 25 aircraft, production rate is of much less interest than when 600 aircraft are involved. Nevertheless, since a 5 percent difference in manufacturing hours is not trivial to an airframe contractor, we attempted to isolate the effect of production rate changes in several aircraft production programs.

Four aircraft (the A-7, F-4, F-102, and KC-135) for which data on manufacturing hours per pound were available, and which had lengthy production runs with a number of changes in production rate, were selected for study. A regression analysis was performed on unit hours per pound against three independent variables: cumulative total production quantity, cumulative model quantity, and lot production rate (see Table 4). Rate appeared to be statistically significant for all four aircraft, but the Durbin-Watson statistic indicated serial correlation of the error term in the F-4, F-102, and KC-135. Moreover, the sign of the production-rate exponent for the KC-135 was positive, suggesting that manufacturing man-hours per pound increase when production rate increases, which is contrary to all assumptions about learning curves.

To eliminate common trends in the variables, first differences of the variables were regressed (Table 5). Because of negative values in the first differences of hours per pound and rate, a linear rather than logarithmic regression was necessary. The coefficients obtained for rate were, therefore, not directly comparable to those above. They were all negative, but serial correlation was still found in the F-4 and KC-135 cases. In terms of statistical parameters, the most reliable results using both the initial data and first differences were obtained for the A-7.

The range of exponents for the production rate variable in all regressions considered (excluding the positive value obtained for the KC-135) suggests that doubling the production rate causes a 3 to 7 percent change in manufacturing hours. Despite the equivocal nature

Table 4

MANUFACTURING HOURS PER POUND AS A FUNCTION OF
TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE
($\text{Ln } Y = \text{Ln } A + B \text{ Ln } X1 + C \text{ Ln } X2 + D \text{ Ln } X3$)

Aircraft Program	Constant	Total Quantity	Model Quantity	Production Rate	
A-7					
Value	12.065	-.066	-.175	-.099	
T-ratio	25.386	-3.084	-7.620	-3.286	
Significance level	.000	.005	.000	.003	
R^2					.918
Standard error (%)					+16, -14
Durbin-Watson					1.524
F-4					
Value	50.315	-.337	-.077	-.049	
T-ratio	50.138	-27.914	-4.547	-1.924	
Significance level	.000	.000	.000	.059	
R^2					.958
Standard error (%)					+12, -11
Durbin-Watson					1.417
F-102					
Value	33.508	-.241	-.109	-.101	
T-ratio	68.287	-8.411	-4.557	-2.974	
Significance level	.000	.000	.000	.005	
R^2					.979
Standard error (%)					+13, -12
Durbin-Watson					1.013
KC-135					
Value	13.440	-.456		.142	
T-ratio	45.888	-33.171		4.948	
Significance level	.000	.000		.000	
R^2					.966
Standard level (%)					+14, -12
Durbin-Watson					.308

Table 5

FIRST DIFFERENCES: MANUFACTURING HOURS PER POUND AS A FUNCTION OF TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE
($\Delta Y = A + B\Delta\ X1 + C\Delta\ X2 + D\Delta\ X3$)

Aircraft Program	Constant	Total Quantity	Model Quantity	Production Rate	
A-7					
Value	.009	-.191	-.098	-.074	
T-ratio	.319	-2.371	-4.841	-2.852	
Significance level	.753	.026	.000	.009	
R^2					.746
Standard error (%)					.121
Durbin-Watson					2.182
F-4					
Value	-.009	-.206	-.083	-.041	
T-ratio	-.452	-2.278	-6.453	-2.021	
Significance level	.653	.026	.000	.048	
R^2					.782
Standard error (%)					.139
Durbin-Watson					3.016
F-102					
Value	-.006	-.146	-.118	-.083	
T-ratio	-.206	-.864	-4.646	-.985	
Significance level	.838	.392	.000	.330	
R^2					.388
Standard error (%)					.124
Durbin-Watson					2.503
KC-135					
Value	-.018	-.193		-.028	
T-ratio	-2.580	-4.871		-1.575	
Significance level	.013	.000		.121	
R^2					.379
Standard error (%)					.040
Durbin-Watson					.970

of some of the evidence, an average value of 5 percent was chosen as representative for the airframe industry.

MATERIALS

Regression analysis of the type described in the previous subsection was used in an attempt to isolate the effect of rate on materials cost. Using data on approximately 1100 A-7A/B/D/Es and the same independent variables used for manufacturing labor hours per pound, the results shown in Table 6 were obtained.

Table 6

MANUFACTURING MATERIALS DOLLARS PER POUND AS A FUNCTION OF TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE
(Ln Y = Ln A + B Ln X1 + C Ln X2 + D Ln X3)

Statistical Values	Constant	Total Quantity	Model Quantity	Production Rate	
Value	42.779	-.057	-.037	-.077	
T-ratio	76.711	-5.235	-3.058	-5.183	
Significance level	.000	.000	.005	.000	
R^2					.898
Standard error (%)					+8, -7

The exponent of the rate variable is -0.077, which translates into a 5 percent change in materials cost for a factor of two change in production rate. Analyses of other programs have produced results of the same magnitude.

OVERHEAD

Overhead rate (the ratio of overhead cost to direct labor cost) is a function of the volume of business a company does, and volume, in turn, reflects rate of output. The fundamental question, however, does not concern production rate, it concerns overall level of business, much of which may be in programs separate from the particular aircraft program of interest. Here we were interested only in determining whether a survey of industry data would indicate a consensus on the degree to which burden rates are a function of direct labor.

An elasticity factor of 0.25 (for example, a 4 percent change in direct labor results in a 1 percent change in overhead rate) was developed in a previous Rand study[5] based on a relatively small sample of companies. The data needed to make similar examination of other companies were not available, but it was possible to examine fluctuations in rate as a function of changes in employment level (employment level serving as a proxy for volume of business). We assumed that overhead rate would be a function of employment and calendar time, but regression analysis of data from four companies did not substantiate that hypothesis (Table 7). For Company A, both employment level and time were statistically significant, and overhead was found to be fairly sensitive to factory employment. Figure 3 shows the data adjusted to the 1974 price level. Actual employment figures are not shown, but the index gives a measure of the relative size of the four companies. Data are for the years 1966-1975.

Employment level was not significant for Company B, and, as shown by Fig. 3, the data are too dispersed to show a definite trend. For Company C both time and employment were significant, while in Company D neither variable was significant. Moreover, for Company D the employment variable had a positive sign, thus implying that overhead is directly proportional to the number of workers.

Changes in accounting procedures can invalidate time-series data, and we know that such a change occurred in Company D. Similar changes may have occurred in Companies B and C, but the smooth relationship shown by Fig. 3 for Company A is an exception. Within limits, management can fix the overhead rate to accord with management objectives (for instance, employment in the Commercial Aircraft Division of the Boeing Company declined from 43,000 in 1970 to 25,000 in 1971, with no increase in factory overhead rate).

A recent (unpublished) study by System Planning Corporation also remarks on the significant variations among companies in their tendencies to hire or lay off indirect labor as the direct labor force goes up or down. Their analysis suggested that in a sample of 10 aircraft

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[5] *Production Rate and Production Cost.*

Table 7

OVERHEAD RATE AS A FUNCTION OF EMPLOYMENT LEVEL AND TIME

	R^2	SEE
Company A		
OH = 215 - .0124E + 2.39T (.025) (.030)	.67	12.1
Company B		
OH = 143.3 -.00024E + 2.08T (.350) (.025)	.55	10.0
Company C		
OH = 132.1 -.0037E + 2.09T (.020) (.025)	.44	12.4
Company D		
OH = 106 + .0021E + .41T (.225) (.80)	.29	10.1

NOTE: OH = factory overhead rate (%); E = factory employment; and T = calendar year minus 1960.

companies a 50 percent reduction in direct labor personnel could be accompanied by a 36 to 59 percent reduction in indirect personnel. A 36 percent decrease would cause the overhead rate to go up perceptibly, but a 59 percent decrease might not affect that rate.

Because overhead rates behave differently from company to company and within the same company from situation to situation, it would be presumptuous to claim that any estimating model has universal validity. In most cases the unit cost of each item produced will be higher when the rate of output is low because each item bears a larger share of the overhead costs. To reflect that fact in the model we have adopted the elasticity factor of 0.25 mentioned above. While it will not be applicable in every instance, it is representative of the nominal change characteristic of the industry.

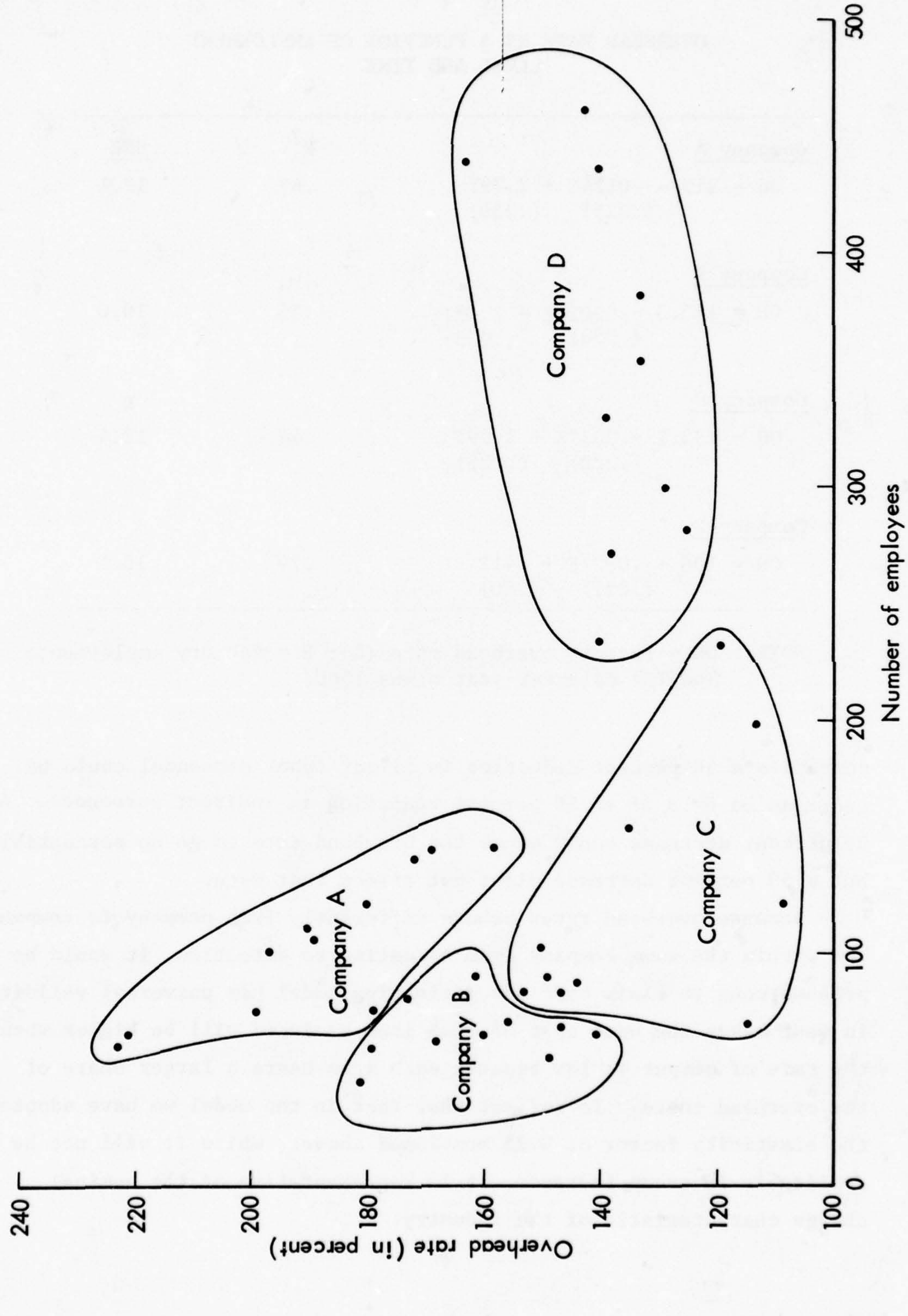

Fig. 3—Index of employment

COST MODEL OUTPUTS

The estimating relationships already described were used with those of DAPCA III[6] to estimate the recurring cost of producing 25 aircraft (units 1-25) with three different delivery periods: 12 months, 24 months, and 48 months. Airframe unit weight and maximum speed are those of the A-10, namely, 14,840 lb and 450 kn. The results (Fig. 4) must be treated with some caution because the 12-month and 48-month cases may be too extreme. As mentioned above, an attempt to deliver units 1-25 of a modern military aircraft in 12 months would probably increase rather than reduce costs, but the model does not reflect that. Similarly, if an acceptance period of 48 months was planned from the outset, management probably could find a way to trim the engineering staff and bring costs down. In effect, the curve could be flatter than indicated in Fig. 4, but probably not steeper.

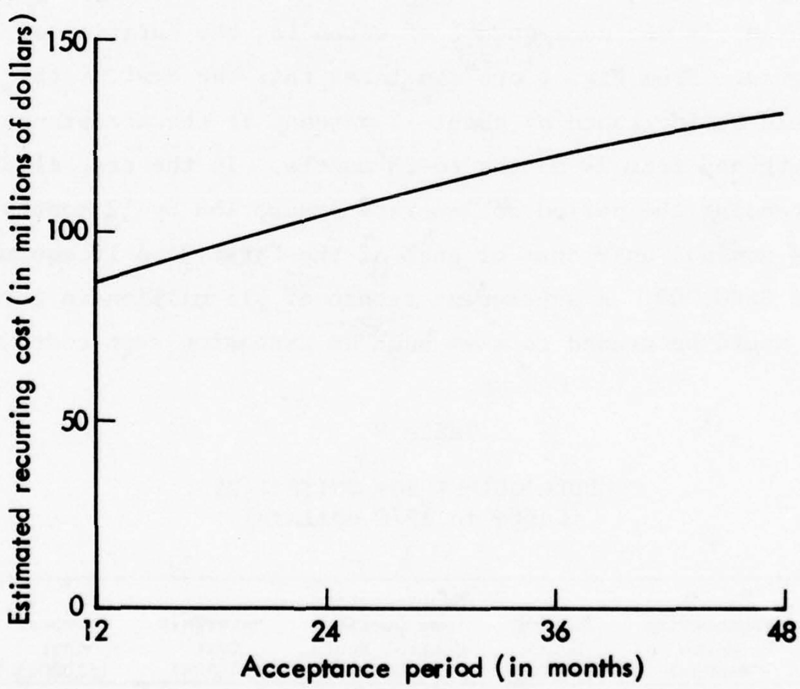

Fig. 4—Estimated recurring cost of aircraft 1-25 versus acceptance period

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[6] H. E. Boren, Jr., *A Computer Model for Estimating Development and Procurement Costs of Aircraft (DAPCA III)*, The Rand Corporation, R-1854-PR, March 1976.

Another reason for caution is that the results are sensitive to the way in which nonrecurring and recurring engineering hours are separated. The model estimates *total* engineering hours (excluding flight test), and to obtain recurring hours it is necessary to assume that the point at which the curve showing cumulative total engineering hours crosses the ordinate (the Y-intercept) represents the nonrecurring portion of the total. That point is determined by the slope of the curve, and small differences in slope translate into large differences in nonrecurring hours. If nonrecurring hours are overstated in the estimate, resulting differences in recurring hours because of different production rates are minimized.

Table 8 shows the detail underlying the totals displayed in Fig. 4. All costs have been reduced to 1970 dollars to ease comparison with the results obtained from the Fairchild and Northrop models. The evaluation indicates that increased cost, primarily in engineering and overhead, is one consequence of extending the duration of a production program. From Fig. 4 one can infer that the cost of the first 25 aircraft would be increased by about 15 percent if the acceptance period were lengthened from 24 months to 36 months. In the case illustrated here, extending the period of low-rate production by 12 months would raise the nominal unit cost of each of the first 25 A-10 equivalents by almost $600,000. A subsequent return of $15 million in total benefits would be needed to make such an extension seem cost effective.

Table 8

MODEL OUTPUT FOR UNITS 1-25
(Costs in 1970 dollars)

Acceptance Period (months)	Engineering Hours (1000s)	Tooling Hours (1000s)	Manufacturing and Quality Control Hours (1000s)	Materials Cost ($1000s)	Overhead Cost ($1000s)	Total Cost ($1000s)
12	756	1444	4784	12,217	44,450	85,900
24	1505	1444	5030	12,887	55,530	103,100
48	2609	1444	5290	13,593	73,600	129,500

That, conceivably, could be provided through modifications not required, more favorable maintenance and operating costs, or better aircraft performance (sortie rate, for example) which could increase unit effectiveness. Two other factors also require mention when considering costs and potential benefits. First, extending an ongoing production program (as the models effectively do) presumably would cause costs to become larger than would be the case if a low-rate program had been optimally planned from the start. Second, extended testing also has associated costs, and these would have to be offset by net benefits if cost effectiveness were to be assured. Additional research is needed to resolve such uncertainties.

IV. THE FAIRCHILD MODEL

The two competing contractors in the A-X program, Northrop and Fairchild, submitted rate variance formulas as part of their development proposals.[1] These formulas were to be used to determine the price for production buys that ranged 50 percent above and below the baseline quantities for Option 1 (26 aircraft) and Option 2 (22 aircraft). The original contract calling for six development test and evaluation (DT&E) and four initial operational test and evaluation (IOT&E) aircraft was abrogated when Congress eliminated the IOT&E lot from the program, but the new agreement retained the original rate variance formulas. New prices were established for the first four production aircraft and the formulas applied to subsequent units.

Option 1 specifies that the delivery period (11 months) will not change whether the quantity is as low as 13 or as high as 39. Thus, the price established by Fairchild for delivery rates ranging from 1.18 per month to 3.55 per month indicates the penalty the Air Force would pay for choosing the lower rate.

The Option 1 formulas are shown in Table 9. The cost categories are defined as follows:

Variable direct labor	=	direct labor, other direct costs, material and purchased parts, and overtime premium
Variable equipment	=	purchased equipment and subcontract
Variable overhead	=	tooling overhead, quality assurance overhead, manufacturing overhead, other overhead
Fixed engineering	=	all recurring engineering costs--direct labor, overhead, overtime premium, material and other direct costs

[1]The information on the A-10 formulas is taken from J. J. Gaunt, Jr., *An Examination of the Rate Variance Formula for the A-10 Air Vehicle*, Air Force Institute of Technology, Wright-Patterson Air Force Base, September 1974.

Table 9

RATE VARIANCE FORMULA FOR OPTION 1

$$\frac{(\leq 26 \text{ aircraft})}{(\text{Total price})} = \left\{ \left(\text{VLAB}_{\text{opt } 1} \right) \frac{M_i}{M_{\text{opt } 1}} + \text{VEQUIP}_i + \text{VOVHD}_{\text{opt } 1} + \text{NRT}_i + \text{FEC}_{\text{opt } 1} \left[.75 + .25 \left(\frac{M_i}{M_{\text{opt } 1}} \right) \right] + \text{G\&A}_{\text{opt } 1} \right\} 1.1$$

$$\frac{(>26 \text{ aircraft})}{(\text{Total price})} = \left\{ \left(\text{VLAB}_{\text{opt } 1} \right) \frac{M_i}{M_{\text{opt } 1}} + \text{VEQUIP}_i + \text{VOVHD}_{\text{opt } 1} + \left(\frac{M_i}{M_{\text{opt } 1}} - 1 \right) (.85) \left(\text{direct } \$_{\text{opt } 1} \right) + \text{NRT}_i + \text{FEC}_{\text{opt } 1} \left[.75 + .25 \left(\frac{M_i}{M_{\text{opt } 1}} \right) \right] \right\} (1.09233)(1.1)$$

NOTE:

M	= manufacturing man-hours
M_i	= manufacturing man-hours for i aircraft
Direct cost	= direct labor cost
VLAB	= variable direct labor costs
VEQUIP	= variable equipment costs
VOVHD	= variable overhead costs
FEC	= fixed engineering costs
NRT	= nonrecurring tooling costs

Nonrecurring tooling	=	all nonrecurring tooling costs
G&A	=	all costs incurred by the executive, legal, financial, marketing, and contracts administration functions. Also includes bid preparation and proposal, independent R&D, and a prorated share of corporate G&A.

The formula for 26 aircraft or less is interesting for several reasons. It levies no penalty on the following three cost categories for reducing the size of the buy: variable direct labor, variable equipment, and nonrecurring tooling. Fixed engineering cost is somewhat biased in favor of the full buy of 26 aircraft, but the penalty for choosing the other option is not substantial. The higher unit cost when buying less than 26 is almost entirely due to variable overhead and G&A, which represent almost 50 percent of total cost. Overhead, the largest single cost category, does not decrease when the buy is reduced. What Fairchild calls "variable" overhead is not variable until the volume of business on contract exceeds the cost of 26 aircraft.

When the buy exceeds 26 aircraft, an additional fixed cost is incurred because additional (rate) tooling is required. The other major changes involve overhead and G&A. Incremental overhead varies as a function of manufacturing man-hours. Instead of being fixed, G&A is a flat 9.283 percent of all other costs.

The rate variance formula can be used to estimate the costs of the first 13 aircraft in buys of 13, 26, and 39 aircraft (Table 10). Nonrecurring tooling has been omitted from the costs in the table, however. An additional $4.8 million for rate tooling would be required for the largest buy, but since that cost would ultimately be incurred in any event (because a high production rate eventually would be approved), it does not change the real cost difference among alternatives.

The rate variance formula indicates that a buy of 13 aircraft would seem disadvantageous primarily because of the method of allocating overhead and G&A costs. The cost penalty incident to buying 13 rather than 26 aircraft would be about 29 percent ($870,000 per

Table 10

ESTIMATED COST OF 13 AIRCRAFT PRODUCED AT THREE DIFFERENT RATES
(In millions of 1970 dollars)

	Delivery rate (aircraft/mo)					
Cost Element	1.2	(% of Total)	2.4	(% of Total)	3.5	(% of Total)
Variable direct labor	6.6	(17)	6.6	(24)	6.6	(26)
Variable equipment	7.6	(20)	7.6	(28)	7.6	(30)
Variable overhead	12.9	(33)	6.4	(23)	5.2	(20)
Fixed engineering	4.2	(11)	2.4	(9)	1.7	(7)
G&A	4.1	(11)	2.1	(8)	2.0	(8)
Fee or profit	3.5	(8)	2.5	(9)	2.3	(9)
Total	38.9		27.6		25.4	

aircraft). The cost benefit of producing at the highest rate--3.5 aircraft per month--comes to about 8 percent ($2.2 million).

Another way to examine Fairchild's treatment of the cost of low production rates is to compare the rate variance formula cost of buying 26 aircraft in Option 1 with that of buying 13 in Option 1 and 13 in Option 2. In the first case the delivery period would be 11 months; in the second, 16 months. The cost penalty for extending the delivery period by five months is said to be $17.3 million:

Option 1 = $53.6 million
Option 1 + Option 2 = $70.9 million

The difference, again, is almost entirely due to the fact that the formula treats overhead and G&A costs as fixed unless the buy exceeds the nominal number of aircraft. These costs are the same whether the total buy is 24 aircraft or 48 aircraft. This portion of the formula seems artificial and unrelated to Fairchild's own experience.

The rate variance formula was developed in advance of production experience with the A-10. But were the relationships between rate and the various functional cost elements implied by the formula comparable to those implicit in cost estimates developed by Fairchild several years later? Based on regression analyses of those estimates

(with production quantity and rate as independent variables), it appears that rate per se is not a statistically significant explanatory variable for either engineering or tooling. It is significant at the 0.14 level for manufacturing hours, but the sign is wrong--rate and hours are related directly rather than inversely. (This is because Fairchild predicts a flattening of the learning curve at the time that production rate increases from 1.83 to 2.5 per month. Thereafter, the slope of the curve remains constant despite further increases in rate until it reaches 16 aircraft per month.) On balance, the regressions appear to corroborate the rate variance formula in that engineering, tooling, and manufacturing hours are not seen to be functions of production rate. Overhead, however, is a function of direct labor hours in the program estimates and is much more variable than indicated in the formula. Consequently, the cost penalties established by strict application of the formula appear to be unreasonably high.

V. THE NORTHROP MODEL

The Northrop computer model for the A-9/A-10 competition[1] incorporates all major recurring cost elements: design engineering, tooling, manufacturing engineering, factory labor, material, other direct costs, and all items associated with overhead. It also incorporates business and economic situations. We investigated different delivery schedules, delivery quantities, and business bases to determine their effects on cost. The model computes labor hours, setups, labor rates, volume purchases, escalation, burden rates, and rate tooling.

The output of the model consists of recurring production costs, system program management and data costs, and, if the production rate exceeds 20 aircraft per month, additional rate tooling costs. It should be noted that the model was not intended to price out very low rate production. The results and the interpretation of the output are useful only to the extent that the model is valid for that condition. Also, the model was designed for use by Northrop and may not apply to other airframe manufacturers.

RESULTS

The model output shows unit cost, by government fiscal year buy, for six years; a breakout of the cost into major cost elements; a breakout of the labor components in hours; and a summary of the rates used (which include overhead). Three questions are of interest: How much will the unit cost increase if production is slowed? How do the individual cost elements behave? What is the increase in unit cost with a decrease in the business base? The latter can be investigated only within narrow limits because of the construction of the model.

BUSINESS BASE

Business base is defined as that business a company has in addition to its contract to develop, test, and produce an aircraft under

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[1]Northrop has developed a more sophisticated model since that competition, so no inferences about current pricing policy are warranted from the data presented here.

the extended schedule. Extended testing and slow initial production could be a hardship on those firms with a small business base. An aircraft producer with a broad business base could shift employees from one project to another in the event of a slowdown, but a single-project company would be forced to lay off personnel in similar circumstances, and the cost effects would be quite different.

We examined the four principal overhead rates that resulted by allowing business base to decline as much as 20 percent (even though that is probably outside the intended limits of the model). The rates considered were engineering, factory (production labor), materials, and G&A. The results are displayed in Table 11.

Table 11

AVERAGE OVERHEAD RATES AS A FUNCTION OF BUSINESS BASE

Overhead Category	Normal	20% Decline
Engineering	1.08	1.12
Factory (production labor)	1.58	1.71
Material	.11	.12
G&A	.18	.19

In the Northrop model, outside business is a very large fraction of total business for those years when the low-revenue development and early production phase of the A-9 program would have occurred. Consequently, declines in the business base affect overhead rates much more than changes (declines) in the production rate of the A-9.

Table 11 indicates that the biggest change in overhead rate would occur in the factory labor category and that engineering overhead rate would change least. This is consistent with expectation because Northrop's outside business is principally production oriented.

Using a weighted average increase (45 percent factory labor, 25 percent engineering labor, 20 percent materials, and 10 percent G&A), the average overall burden rate increase would be 3.95 percent for a 20 percent decline in the business base.

PRODUCTION RATE

The Northrop model indicates that for a one-year production period the unit cost for the first 6 production airframes (following 10 test aircraft) would be nearly twice the unit cost of each airframe if 24 were produced. Another way of looking at the cost of early production units is to compute the total cost of an equivalent number of aircraft (in this case 24) at different rates. The result is shown in Table 12.

Table 12

PRODUCTION COST OF 24 AIRFRAMES

Rate (Units/Month)	Time (Years)	Total Cost ($ millions)	Cost/Unit ($ millions)
1/2	4	69	2.9
1	2	53	2.2
2	1	45	1.9

Of course, there are some time effects. For example, some of the costs tend to be more or less fixed by the size of the staff. Engineers are regarded as less fungible than production workers. Thus, if the duration of a project is extended, engineering costs increase. Table 13 more closely evaluates this effect.

Table 13

COST ELEMENTS OF 24 AIRFRAME UNITS

Cost Element	Production Rate (Units/Month)		
	1/2	1	2
Engineering hours (millions)	1.848	1.131	.739
Tooling hours (millions)	.440	.268	.174
Factory hours (millions)	2.158	2.044	2.039
Material ($ millions)	10.88	10.51	10.31

Table 13 shows that engineering and tooling costs are very time-dependent, whereas factory hours and material costs are nearly independent of the length of time required to produce the first buy of aircraft. Tooling hours, shown in Table 13, require some explanation. Northrop based the A-9 tooling rate variance formulas on T-38 experience. The T-38 program tooling followed a 65 percent curve (very similar to that for engineering which, for the A-9, is on a 59 percent curve). Thus for the A-9 calculation, tooling, like engineering, appears to constitute a fairly substantial effort regardless of the rate of output of aircraft. (Current Northrop practice is to treat tooling more like factory labor, which means that the Northrop tooling effort would more closely resemble the tooling for the Fairchild and Rand model outputs.)

As seen in Table 13, there is a 5.8 percent difference between factory labor costs at the production rate of 6 per year and a rate of 24 per year. Current Northrop thinking is that 10 percent would be a more realistic figure.

OVERALL COST IMPACTS

The principal cost of a reduced rate early in a production program appears to result from the increased length of the program. Cost effects have been evaluated by examining four hypothetical programs, three with early-phase low production rates, and one built around a normal, higher rate throughout the program lifetime. In all four cases, the assumed total buy is 600 aircraft (about the size of the A-10 program). This 6-year production period is the maximum permitted by the Northrop model. Such hypothetical programs do not consider time-phasing funds, annual funding limitations, long-lead-time items, or any of the other realities of life in aircraft production.

The most striking result is that it does not seem to make much difference, in a 6-year production run, how low the initial production rate is (provided that the business base is sufficiently high to carry the fixed costs on other programs). Initially, of course, low production rates do increase aircraft costs. Given a reasonable business base, the cost of extending the production period for the first lot of

24 A-9s would have been about $8 million per year in 1970 dollars (or about $13 million in 1977 dollars). The additional cost is attributed primarily to engineering and overhead. However, although additional costs are incurred because of the very low rate of early output, the higher cost of early low production rates are entirely offset by the cost benefits of subsequent higher production rates. It must be emphasized that both of the preceding statements assume a substantial fixed business base. A major reduction in business base during a low production period would increase costs substantially. Nevertheless, in a "best case" setting, any benefits that accrued to such a program because of an extended period for testing and incorporating changes would be cost free, at least in terms of total production cost.

VI. COMPARISON OF RESULTS

The Fairchild and Northrop rate variance formulas exclude costs of a prototype and 10 full-scale development aircraft, so a direct comparison with the Rand model can be made only for production aircraft. The comparisons below, then, are based on 24 aircraft produced at different rates after production of all test aircraft. The Rand and Northrop models can be compared directly because any production rate can be considered (although rates of less than one month are outside the range intended for the Northrop model). The Fairchild model examines a more limited set of production rates, but permits inferences from the formula itself about the effect of changing lot size while holding delivery period constant.

ENGINEERING

In the Fairchild rate variance formula, 75 percent of the engineering hours are constant regardless of whether 13 or 39 aircraft are delivered. This argues that over a given period--in this case 11 months--production rate is largely irrelevant in determining engineering cost. Pushing the Fairchild premise to an extreme, one could infer that engineering hours vary directly with time: a 48-month delivery schedule would require four times as many engineering hours as a 12-month schedule. The size of the engineering staff is not that inflexible, of course, but some minimum has to be established. A project staff is maintained to handle service complaints, modifications, redesign, etc., after the airplane is in field service. Also, engineers are categorized by specialties, such as aerodynamics, structure, propulsion, and electronics, so that even if a minimum staff were desired, the engineering total still must contain each specialty. The Northrop model showed that engineering cost is basically time-dependent; a significant portion of engineering operates at a fixed level of about one-half million hours per year (about 250 engineers). The cost of program extension, then, would be of that magnitude. That

estimate is for a relatively small and simple airframe, similar to the A-9 or A-10. A larger engineering staff would be required for both larger and more complex airplanes.

As shown in Table 14, the Rand model predicts a smaller increase in engineering hours than does the Northrop model, but the difference is not great.

Table 14

EFFECT OF PROGRAM EXTENSION ON ENGINEERING HOURS

Months to Deliver 24 Aircraft	Engineering Hour Ratios	
	Northrop	Rand
12	1.00	1.00
24	1.53	1.47
48	2.50	2.17

Thus all three models support the notion that engineering is largely a level-of-effort endeavor and hence is closely related to program length.

TOOLING

Only an initial set of tools would be required for low-rate production. Nonrecurring tooling would not be affected by rate changes of the magnitude considered here, and sustaining tooling is generally treated as a function of factory hours. Tooling hours are included with manufacturing hours in the Fairchild model and cannot be shown separately, but the intent is clearly to have them vary with production quantity, not rate. The Northrop A-9 model treats tooling differently, but as explained previously, the model currently in use at Northrop conforms more closely to normal practice. The Rand model treats tooling as fixed for a fixed quantity of aircraft. Although, admittedly, some portion of tooling is time-dependent, statistically that portion could not be isolated, and we believe that within the limits of the rates examined production rate has little effect on tooling hours.

MANUFACTURING LABOR

As shown in Table 15, Northrop shows an increase of 6 percent in direct manufacturing hours when program length is increased from 12 months to 48 months. The Rand model would show a 10 percent increase, and that figure is supported by the current Northrop model. The direct labor cost element in the Fairchild model (VLAB) includes a variety of subelements including materials and purchased parts, so a direct comparison with the other models is not possible. One can infer that on the basis of the rate variance formula, manufacturing labor varies only as a function of quantity because the slope of the learning curve is constant whether 13 or 39 aircraft are delivered in an 11-month period. Thus all three models imply that rate has a minor effect on direct manufacturing hours.

Table 15

EFFECT OF PROGRAM EXTENSION ON MANUFACTURING LABOR HOURS

Months to Deliver 24 Aircraft	Manufacturing Labor Ratios	
	Northrop	Rand
12	1.00	1.00
24	1.00	1.05
48	1.06	1.10

MANUFACTURING MATERIALS

Materials cost is influenced by volume purchases, so one would expect to see the cost of materials decrease as the production rate increases. As shown in Table 16, the Northrop model indicates that there is only a 5 percent increase in materials cost with a factor-of-four decrease in rate. The Rand model shows a 5 percent increase for a factor-of-two change. Fairchild combines materials with direct labor, but separately estimates purchased equipment, normally treated as part of materials cost. Fairchild's purchased equipment category did not vary with production rate. Thus materials are essentially unaffected by production rate changes.

Table 16

EFFECT OF PROGRAM EXTENSION ON MATERIALS COST

Months to Deliver 24 Aircraft	Materials Cost Ratios	
	Northrop	Rand
12	1.00	1.00
24	1.02	1.05
48	1.06	1.10

OVERHEAD

The various categories of indirect cost--engineering and production overhead, material overhead, and G&A--may account for over half the cost of a stretched production program. These costs are, therefore, the most important to consider when planning an extended development-production transition. Because of its arbitrary treatment of overhead costs, the Fairchild model provides no clues as to what effects an increase in these might create, but the Northrop and Rand models agree that increasing the delivery period from 12 months to 24 months will increase overhead cost by 27 to 36 percent. Northrop's calculations indicate that stretching the program to 48 months would double the cost. The Rand model predicts an increase of only about 60 percent (see Table 17). Differences of that magnitude can stem from different assumptions about business base and fixed costs, but both estimates imply that a contractor will act to keep indirect costs down when the volume of production is low.

Table 17

EFFECT OF PROGRAM EXTENSION ON OVERHEAD COSTS

Months to Deliver 24 Aircraft	Overhead Cost Ratios	
	Northrop	Rand
12	1.00	1.00
24	1.36	1.27
48	2.05	1.61

TOTAL COST

Figure 5 displays curves based on estimates, using all three models, of the total cost of 24 aircraft. Beyond 16 months, the Fairchild curve is an extrapolation; it is included because it may represent an upper bound on the cost increase that could result from a lengthy development-production transition. The Northrop and Rand curves are almost congruent for the period from 12 to 24 months, indicating perhaps that an 18 percent increase in cost would be the probable price of maintaining a production rate of one aircraft per month over a 24-month period. The cost penalty of reducing the rate to 0.5 aircraft per month will be 40 to 50 percent.

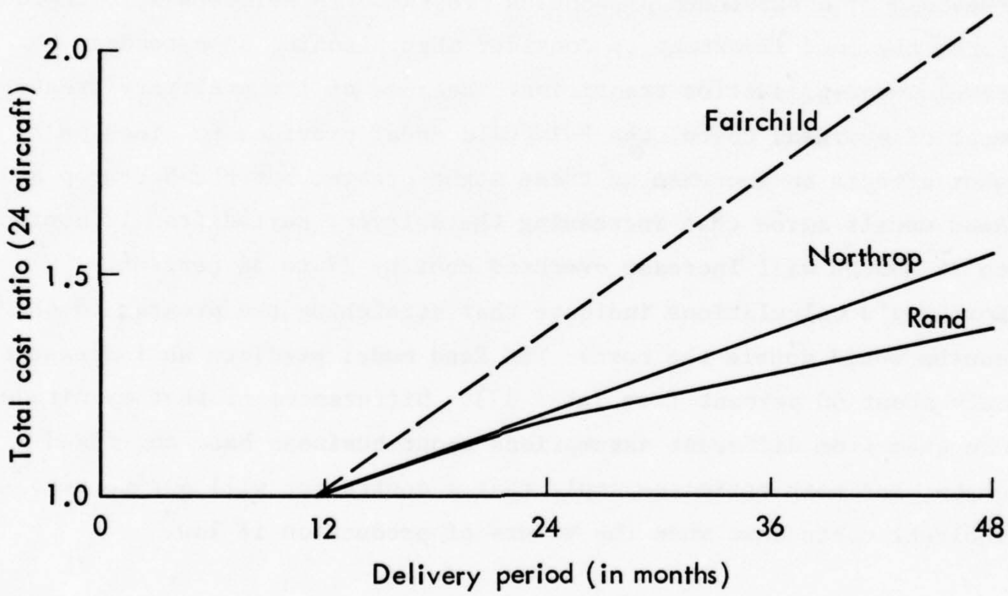

Fig. 5— Effect of program extension on total cost of 24 aircraft

INFLATION

At current inflation rates, a delay of one to three years in starting full-scale production could cause program costs to increase substantially. But from a theoretical point of view price-level changes resulting from inflation should not be a consideration in deciding how to conduct a program. The important factors are real resource costs: man-hours, materials, tools, etc. These will not be changed by inflation. Given that a time preference for money exists, as exemplified by the 10 percent discount rate used in many DoD studies, it would be theoretically advantageous to postpone major expenditures for several years. Yet because of continuous pressure to reduce defense spending, it may be more sensible for the Air Force to assume that procurement budgets will not keep pace with inflation. The issue is primarily judgmental. In current planning and budgeting procedures, the services are allowed to project fairly realistic inflation rates for future procurement, which should diminish possible adverse reactions to proposals for using low-initial-rate production as an element of a phased acquisition strategy. In any event, if deliberate extensions of low-initial-rate production are kept to the 18- to 24-month limits that seen to be warranted by recent experience, neither inflation nor time-preference discounting becomes a major consideration.

VIII. CONCLUSIONS

There are both advantages and disadvantages to extending the transition period between development and production of airframes. The disadvantage most commonly cited is the additional cost incurred by underutilizing a contractor's facilities, equipment, and personnel. Our examination of cost factors suggests that the incremental cost of extended initial-low-rate production has often been overstated because it cannot be easily distinguished from other forms of cost growth. Yet it remains true that when all other elements of life cycle cost are disregarded, the production cost of aircraft is higher when they are produced at very low rates. The models described here suggest that the cost penalty will be about 18 to 38 percent of the cost of the first 24 aircraft, although the business base of the contractor is a dominating consideration.

In the context of life cycle cost, one can anticipate a number of offsetting savings. Where the business base is sufficiently high to carry the fixed costs, increased costs in the early years may be offset by the lower costs of producing later aircraft at an efficient high rate. Savings in retrofit modifications would accrue; in some programs the costs of such modifications have been very high. Also, when a low production rate is planned from the outset, rather than being imposed after a program has started, costs can be controlled much better. A premature buildup of the direct and indirect personnel needed for high-rate production can be avoided. A prudent planner should anticipate and budget for higher costs during an extended development-production phase, but there is a good possibility that some or all of those funds will be recouped later. Consequently, unit production cost alone should not be the dominant criterion in decisions involving an extended development-production transition.

Appendix

RESULTS OF REGRESSION ANALYSIS

Table A-1

ENGINEERING HOURS VS AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD - AIRCRAFT 1-25

NOTE — STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		0.10079D-02				
LN A	(CONSTANT)	-6.89988	1.77050	-3.89715	0.00089	
B	X1	0.71277	0.07927	8.99193	0.00000	0.91163
C	X2	1.05071	0.18840	5.57688	0.00002	0.56617
D	X3	0.43975	0.19718	2.23023	0.03735	0.20883

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	8.60663	0.78893	1.00000	0.70062	0.22615	0.28577
LN X1	10.09845	1.00904	0.70062	1.00000	-0.38840	0.04259
LN X2	6.56283	0.42511	0.22615	-0.38840	1.00000	0.06732
LN X3	3.21317	0.37465	0.28577	0.04259	0.06732	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.82642	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.02960
STANDARD ERROR OF ESTIMATE	0.35249	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.04096
SUM OF SQUARES OF RESIDUALS (LN)	2.48492	SUM OF SQUARES TOTAL (LN)	14.31557
F VALUE	31.73988	DURBIN-WATSON STATISTIC	1.78076
DEGREES OF FREEDOM FOR ERROR	20	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	23	NUMBER OF DATA POINTS	24

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.31347D 01	-0.53579D-01	-0.19617D 00	-0.12367D 00
B	-0.53579D-01	0.62834D-02	0.58620D-02	-0.11686D-02
C	-0.19617D 00	0.58620D-02	0.35496D-01	-0.33839D-02
D	-0.12367D 00	-0.11686D-02	-0.33839D-02	0.38879D-01

Table A-2

ENGINEERING HOURS VS AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD - AIRCRAFT 26-50

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		0.20288D-02				
LN A	(CONSTANT)	-6.20030	2.14816	-2.88633	0.00913	
B	X1	0.58647	0.10571	5.54785	0.00002	0.67614
C	X2	0.91083	0.23471	3.88069	0.00093	0.44241
D	X3	0.53794	0.15333	3.50847	0.00221	0.39494

VARIABLE	MEAN	STANDARD DEVIATION	CORRELATION MATRIX LN Y	LN X1	LN X2	LN X3
LN Y	6.62764	0.87522	1.00000	0.64618	0.20667	0.66794
LN X1	10.09845	1.00904	0.64618	1.00000	-0.38840	0.35924
LN X2	6.56283	0.42511	0.20667	-0.38840	1.00000	0.06804
LN X3	1.72479	0.64256	0.66794	0.35924	0.06804	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.79214	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.04492
STANDARD ERROR OF ESTIMATE	0.42791	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.06456
SUM OF SQUARES OF RESIDUALS (LN)	3.66218	SUM OF SQUARES TOTAL (LN)	17.61824
F VALUE	25.40573	DURBIN-WATSON STATISTIC	2.00694
DEGREES OF FREEDOM FOR ERROR	20	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	23	NUMBER OF DATA POINTS	24

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.46146D 01	-0.78963D-01	-0.28911D 00	-0.33258D-01
B	-0.78963D-01	0.11175D-01	0.11002D-01	-0.67993D-02
C	-0.28911D 00	0.11002D-01	0.55088D-01	-0.86861D-02
D	-0.33258D-01	-0.67993D-02	-0.86861D-02	0.23509D-01

Table A-3

ENGINEERING HOURS VS AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD - AIRCRAFT 51-100

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A	0.91579D-04				
LN A (CONSTANT)	-9.29831	2.29479	-4.05192	0.00068	
B X1	0.72171	0.11017	6.55070	0.00000	0.79622
C X2	1.26476	0.26532	4.76691	0.00013	0.55882
D X3	0.19180	0.17270	1.11061	0.28060	0.13234

			CORRELATION MATRIX			
VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	6.67334	0.89479	1.00000	0.67093	0.34583	0.54681
LN X1	10.03859	0.98717	0.67093	1.00000	-0.30750	0.35166
LN X2	6.59889	0.39535	0.34583	-0.30750	1.00000	0.24064
LN X3	1.98481	0.61739	0.54681	0.35166	0.24064	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.79982	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.04431
STANDARD ERROR OF ESTIMATE	0.43079	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.06455
SUM OF SQUARES OF RESIDUALS (LN)	3.52595	SUM OF SQUARES TOTAL (LN)	17.61429
F VALUE	25.30559	DURBIN-WATSON STATISTIC	2.53535
DEGREES OF FREEDOM FOR ERROR	19	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	22	NUMBER OF DATA POINTS	23

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.52661D 01	-0.86893D-01	-0.35613D 00	-0.43923D-01
B	-0.86893D-01	0.12138D-01	0.12615D-01	-0.87689D-02
C	-0.35613D 00	0.12615D-01	0.70395D-01	-0.17941D-01
D	-0.43923D-01	-0.87689D-02	-0.17941D-01	0.29825D-01

Table A-4

ENGINEERING HOURS VS AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD - AIRCRAFT 101-200

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		0.11323D-02				
LN A	(CONSTANT)	-6.78352	2.57769	-2.63163	0.01972	
B	X1	0.68798	0.13131	5.23938	0.00013	0.83432
C	X2	0.95237	0.29231	3.25811	0.00572	0.47759
D	X3	0.15249	0.20948	0.72793	0.47865	0.10918

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	6.56244	0.76181	1.00000	0.71646	0.19614	0.43084
LN X1	9.86787	0.92386	0.71646	1.00000	-0.33613	0.39079
LN X2	6.55390	0.38203	0.19614	-0.33613	1.00000	-0.00918
LN X3	2.06809	0.54543	0.43084	0.39079	-0.00918	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.73847	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.04549
STANDARD ERROR OF ESTIMATE	0.42931	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.06542
SUM OF SQUARES OF RESIDUALS (LN)	2.58025	SUM OF SQUARES TOTAL (LN)	9.86599
F VALUE	13.17707	DURBIN-WATSON STATISTIC	2.93447
DEGREES OF FREEDOM FOR ERROR	14	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	17	NUMBER OF DATA POINTS	18

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.66445D 01	-0.12534D 00	-0.48685D 00	-0.75367D-01
B	-0.12534D 00	0.17242D-01	0.13867D-01	-0.11324D-01
C	-0.48685D 00	0.13867D-01	0.85443D-01	-0.86299D-02
D	-0.75367D-01	-0.11324D-01	-0.86299D-02	0.43883D-01

Table A-5

TOOLING HOURS VS AIRFRAME UNIT WEIGHT, SPEED, AND DELIVERY PERIOD - AIRCRAFT 1-25

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A	0.62692D-01				
LN A (CONSTANT)	-2.76953	2.11502	-1.30946	0.20229	
B X1	0.69192	0.10371	6.67138	0.00000	0.83315
C X2	0.52745	0.24218	2.17791	0.03905	0.27998
D X3	0.25359	0.19986	1.26885	0.21618	0.15289

VARIABLE	MEAN	STANDARD DEVIATION	CORRELATION MATRIX LN Y	LN X1	LN X2	LN X3
LN Y	8.43341	0.78874	1.00000	0.74492	0.02192	0.28328
LN X1	10.05292	0.94974	0.74492	1.00000	-0.35517	0.07329
LN X2	6.55820	0.41867	0.02192	-0.35517	1.00000	0.24763
LN X3	3.10732	0.47553	0.28328	0.07329	0.24763	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.67008	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.04218
STANDARD ERROR OF ESTIMATE	0.47946	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.05685
SUM OF SQUARES OF RESIDUALS (LN)	5.74695	SUM OF SQUARES TOTAL (LN)	17.41920
F VALUE	16.92528	DURBIN-WATSON STATISTIC	1.80517
DEGREES OF FREEDOM FOR ERROR	25	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	28	NUMBER OF DATA POINTS	29

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.44733D 01	-0.91501D-01	-0.30717D 00	-0.11282D 00
B	-0.91501D-01	0.10757D-01	0.97045D-02	-0.36903D-02
C	-0.30717D 00	0.97045D-02	0.58652D-01	-0.14208D-01
D	-0.11282D 00	-0.36903D-02	-0.14208D-01	0.39945D-01

Table A-6

A-7 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATES

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A	12.06504				
LN A (CONSTANT)	2.49031	0.09810	25.38597	0.00000	
B X1	-0.65845D-01	0.02135	-3.08404	0.00493	-0.22609
C X2	-0.17526	0.02300	-7.62033	0.00000	-0.63635
D X3	-0.99293D-01	0.03022	-3.28550	0.00301	-0.23621

VARIABLE	MEAN	STANDARD DEVIATION	CORRELATION MATRIX LN Y	LN X1	LN X2	LN X3
LN Y	1.22338	0.48327	1.00000	-0.72310	-0.92050	-0.71644
LN X1	5.62782	1.65933	-0.72310	1.00000	0.62470	0.42117
LN X2	4.19516	1.75465	-0.92050	0.62470	1.00000	0.60502
LN X3	1.62250	1.14965	-0.71644	0.42117	0.60502	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.91848	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.10000
STANDARD ERROR OF ESTIMATE	0.14603	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.11936
SUM OF SQUARES OF RESIDUALS (LN)	0.53311	SUM OF SQUARES TOTAL (LN)	6.53927
F VALUE	93.88518	DURBIN-WATSON STATISTIC	1.52384
DEGREES OF FREEDOM FOR ERROR	25	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	28	NUMBER OF DATA POINTS	29

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.96232D-02	-0.15567D-02	-0.10377D-02	-0.93492D-03
B	-0.15567D-02	0.45584D-03	-0.25152D-03	-0.44846D-04
C	-0.10377D-02	-0.25152D-03	0.52898D-03	-0.33557D-03
D	-0.93492D-03	-0.44846D-04	-0.33557D-03	0.91335D-03

Table A-7

FIRST DIFFERENCES: A-7 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATES

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A (CONSTANT)	0.92453D-02	0.02902	0.31854	0.75283	
B X1	-0.19111	0.08061	-2.37073	0.02613	-0.26429
C X2	-0.98082D-01	0.02026	-4.84087	0.00006	-0.54170
D X3	-0.73505D-01	0.02577	-2.85211	0.00879	-0.33316

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	Y	X1	X2	X3
Y	-0.06032	0.22631	1.00000	-0.52032	-0.73270	-0.63646
X1	0.23079	0.31298	-0.52032	1.00000	0.24477	0.37052
X2	0.20277	1.24989	-0.73270	0.24477	1.00000	0.37913
X3	0.07582	1.02575	-0.63646	0.37052	0.37913	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.74646	MEAN OF ABSOLUTE RELATIVE DEVIATIONS	1.06448
STANDARD ERROR OF ESTIMATE	0.12087	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	-2.00366
SUM OF SQUARES OF RESIDUALS	0.35060	SUM OF SQUARES TOTAL	1.38234
F VALUE	23.55348	DURBIN-WATSON STATISTIC	2.18200
DEGREES OF FREEDOM FOR ERROR	24	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	27	NUMBER OF DATA POINTS	28

VARIANCE-COVARIANCE MATRIX

	A	B	C	D
A	0.84237D-03	-0.12748D-02	-0.70227D-04	-0.38987D-04
B	-0.12748D-02	0.64982D-02	-0.19818D-03	-0.64309D-03
C	-0.70227D-04	-0.19818D-03	0.41052D-03	-0.16725D-03
D	-0.38987D-04	-0.64309D-03	-0.16725D-03	0.66420D-03

Table A-8

F-4 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		50.31472				
LN A	(CONSTANT)	3.91830	0.07815	50.13787	0.00000	
F	X1	-0.33651	0.01206	-27.91393	0.00000	-0.81160
C	X2	-0.76983D-01	0.01693	-4.54708	0.00003	-0.23472
D	X3	-0.48984D-01	0.02547	-1.92356	0.05932	-0.10009

			CORRELATION MATRIX			
VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	1.37933	0.55474	1.00000	-0.93117	-0.60522	-0.60085
LN X1	6.22070	1.33795	-0.93117	1.00000	0.35141	0.37053
LN X2	4.59426	1.69143	-0.60522	0.35141	1.00000	0.85224
LN X3	1.87790	1.13349	-0.60085	0.37053	0.85224	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.95793	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.07590
STANDARD ERROR OF ESTIMATE	0.11669	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.08460
SUM OF SQUARES OF RESIDUALS (LN)	0.78978	SUM OF SQUARES TOTAL (LN)	18.77225
F VALUE	440.20018	DURBIN-WATSON STATISTIC	1.41651
DEGREES OF FREEDOM FOR ERROR	58	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	61	NUMBER OF DATA POINTS	62

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.61075D-02	-0.77572D-03	-0.35847D-03	-0.32628D-03
B	-0.77572D-03	0.14533D-03	-0.14964D-04	-0.44531D-04
C	-0.35847D-03	-0.14964D-04	0.28663D-03	-0.35797D-03
D	-0.32628D-03	-0.44531D-04	-0.35797D-03	0.64847D-03

Table A-9

FIRST DIFFERENCES: F-4 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A (CONSTANT)	-0.92008D-02	0.02034	-0.45246	0.65266	
B X1	-0.20574	0.09031	-2.27811	0.02649	-0.14183
C X2	-0.82815D-01	0.01283	-6.45273	0.00000	-0.68187
D X3	-0.41210D-01	0.02039	-2.02090	0.04800	-0.21439

VARIABLE	MEAN	STANDARD DEVIATION	CORRELATION MATRIX Y	X1	X2	X3
Y	-0.03975	0.28973	1.00000	-0.19184	-0.86157	-0.78061
X1	0.10987	0.19973	-0.19184	1.00000	0.04258	0.09783
X2	0.07144	2.38552	-0.86157	0.04258	1.00000	0.81006
X3	0.04911	1.50724	-0.78061	0.09783	0.81006	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.78204	MEAN OF ABSOLUTE RELATIVE DEVIATIONS	0.96194
STANDARD ERROR OF ESTIMATE	0.13878	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	-3.49170
SUM OF SQUARES OF RESIDUALS	1.09778	SUM OF SQUARES TOTAL	5.03655
F VALUE	68.17112	DURBIN-WATSON STATISTIC	3.01632
DEGREES OF FREEDOM FOR ERROR	57	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	60	NUMBER OF DATA POINTS	61

VARIANCE-COVARIANCE MATRIX

	A	B	C	D
A	0.41353D-03	-0.88401D-03	-0.40298D-05	-0.69389D-05
B	-0.88401D-03	0.81563D-02	0.72845D-04	-0.19914D-03
C	-0.40298D-05	0.72845D-04	0.16471D-03	-0.21212D-03
D	-0.69389D-05	-0.19914D-03	-0.21212D-03	0.41583D-03

Table A-10

F-102 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, PRODUCTION QUANTITY, AND PRODUCTION RATE

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		33.50807				
LN A	(CONSTANT)	3.51179	0.05143	68.28695	0.00000	
B	X1	-0.24145	0.02871	-8.41080	0.00000	-0.56897
C	X2	-0.10890	0.02390	-4.55652	0.00004	-0.26303
D	X3	-0.10137	0.03408	-2.97410	0.00463	-0.18101

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	1.90679	0.81502	1.00000	-0.97948	-0.95032	-0.94693
LN X1	4.24728	1.92060	-0.97948	1.00000	0.92115	0.92937
LN X2	3.37342	1.96861	-0.95032	0.92115	1.00000	0.90158
LN X3	2.09299	1.45537	-0.94693	0.92937	0.90158	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.97865	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.05025
STANDARD ERROR OF ESTIMATE	0.12282	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.06441
SUM OF SQUARES OF RESIDUALS (LN)	0.70896	SUM OF SQUARES TOTAL (LN)	33.21317
F VALUE	718.28557	DURBIN-WATSON STATISTIC	1.01706
DEGREES OF FREEDOM FOR ERROR	47	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	50	NUMBER OF DATA POINTS	51

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.26447D-02	-0.34737D-03	-0.26261D-03	-0.29811D-03
B	-0.34737D-03	0.82407D-03	-0.35762D-03	-0.57456D-03
C	-0.26261D-03	-0.35762D-03	0.57115D-03	-0.25793D-03
D	-0.29811D-03	-0.57456D-03	-0.25793D-03	0.11617D-02

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Table A-11

FIRST DIFFERENCES: F-102 MANUFACTURING HOURS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE

PARAMETER	VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A (CONSTANT)	-0.59049D-02	0.02871	-0.20570	0.83798	
B X1	-0.14598	0.16887	-0.86443	0.39204	-0.10575
C X2	-0.11752	0.02530	-4.64550	0.00003	-0.56029
D X3	-0.83169D-01	0.08443	-0.98501	0.33000	-0.12124

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	Y	X1	X2	X3
Y	-0.04463	0.15347	1.00000	-0.21699	-0.59772	-0.24901
X1	0.13754	0.11117	-0.21699	1.00000	0.14551	0.24512
X2	0.12243	0.73167	-0.59772	0.14551	1.00000	0.18179
X3	0.05118	0.22371	-0.24901	0.24512	0.18179	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.38803	MEAN OF ABSOLUTE RELATIVE DEVIATIONS	1.88579
STANDARD ERROR OF ESTIMATE	0.12408	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	-2.78029
SUM OF SQUARES OF RESIDUALS	0.67740	SUM OF SQUARES TOTAL	1.10693
F VALUE	9.29984	DURBIN-WATSON STATISTIC	2.50259
DEGREES OF FREEDOM FOR ERROR	44	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	47	NUMBER OF DATA POINTS	48

VARIANCE-COVARIANCE MATRIX

	A	B	C	D
A	0.82408D-03	-0.36455D-02	-0.74910D-04	-0.33501D-03
B	-0.36455D-02	0.28519D-01	-0.45236D-03	-0.32049D-02
C	-0.74910D-04	-0.45236D-03	0.63996D-03	-0.32541D-03
D	-0.33501D-03	-0.32049D-02	-0.32541D-03	0.71292D-02

Table A-12

KC-135 MANUFACTURING HOURS/POUND VS QUANTITY AND PRODUCTION RATE

NOTE -- STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		13.44031				
LN A	(CONSTANT)	2.59826	0.05662	45.88753	0.00000	
B	X1	-0.45641	0.01376	-33.17149	0.00000	-1.07545
C	X2	0.14724	0.02976	4.94786	0.00001	0.16041

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2
LN Y	0.67699	0.67342	1.00000	-0.97470	-0.51500
LN X1	4.83212	1.58680	-0.97470	1.00000	0.62803
LN X2	1.92978	0.73366	-0.51500	0.62803	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.96563	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.44091
STANDARD ERROR OF ESTIMATE	0.12714	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.18781
SUM OF SQUARES OF RESIDUALS (LN)	0.87292	SUM OF SQUARES TOTAL (LN)	25.39581
F VALUE	758.50816	DURBIN-WATSON STATISTIC	0.30805
DEGREES OF FREEDOM FOR ERROR	54	DEGREES OF FREEDOM DUE TO REGRESSION	2
TOTAL DEGREES OF FREEDOM	56	NUMBER OF DATA POINTS	57

VARIANCE-COVARIANCE MATRIX

	LN A	B	C
LN A	0.32061D-02	-0.55397D-03	-0.10349D-02
B	-0.55397D-03	0.18931D-03	-0.25715D-03
C	-0.10349D-02	-0.25715D-03	0.88558D-03

Table A-13

FIRST DIFFERENCES: KC-135 MANUFACTURING HOURS/POUND VS QUANTITY AND PRODUCTION RATE

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A	(CONSTANT)	-0.17833D-01	0.00691	-2.58025	0.01268	
B	X1	-0.19299	0.03962	-4.87111	0.00001	-0.54589
C	X2	-0.27727D-01	0.01761	-1.57494	0.12122	-0.17650

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	Y	X1	X2
Y	-0.03996	0.05003	1.00000	-0.59164	-0.31800
X1	0.11185	0.14152	-0.59164	1.00000	0.25922
X2	0.01962	0.31848	-0.31800	0.25922	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.37910	MEAN OF ABSOLUTE RELATIVE DEVIATIONS	1.22618
STANDARD ERROR OF ESTIMATE	0.04016	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	-1.00492
SUM OF SQUARES OF RESIDUALS	0.08548	SUM OF SQUARES TOTAL	0.13767
F VALUE	16.17997	DURBIN-WATSON STATISTIC	0.97030
DEGREES OF FREEDOM FOR ERROR	53	DEGREES OF FREEDOM DUE TO REGRESSION	2
TOTAL DEGREES OF FREEDOM	55	NUMBER OF DATA POINTS	56

VARIANCE-COVARIANCE MATRIX

	A	B	C
A	0.47764D-04	-0.16378D-03	-0.56719D-05
B	-0.16378D-03	0.15697D-02	-0.18081D-03
C	-0.56719D-05	-0.18081D-03	0.30994D-03

Table A-14

A-7 MATERIAL DOLLARS/POUND VS TOTAL QUANTITY, MODEL QUANTITY, AND PRODUCTION RATE

NOTE — STATISTICS ARE BASED ON LOGARITHMS

PARAMETER		VALUE	STANDARD ERROR	T-RATIO	SIGNIF LEVEL	BETA COEFF
A		42.77910				
LN A	(CONSTANT)	3.75605	0.04896	76.71061	0.00000	
B	X1	-0.57148D-01	0.01092	-5.23461	0.00002	-0.43883
C	X2	-0.36821D-01	0.01204	-3.05812	0.00525	-0.28849
D	X3	-0.77280D-01	0.01491	-5.18345	0.00002	-0.41006

CORRELATION MATRIX

VARIABLE	MEAN	STANDARD DEVIATION	LN Y	LN X1	LN X2	LN X3
LN Y	3.15446	0.21671	1.00000	-0.79749	-0.81504	-0.76416
LN X1	5.62337	1.66406	-0.79749	1.00000	0.64809	0.41869
LN X2	4.20584	1.69789	-0.81504	0.64809	1.00000	0.59052
LN X3	1.62218	1.14990	-0.76416	0.41869	0.59052	1.00000

COEFFICIENT OF DETERMINATION (UNADJ), R SQ	0.89845	MEAN OF ABSOLUTE RELATIVE DEVIATIONS (LN)	0.01636
STANDARD ERROR OF ESTIMATE	0.07308	COEFF VARIATION (STD ERR EST / MEAN Y OBS)	0.02317
SUM OF SQUARES OF RESIDUALS (LN)	0.13353	SUM OF SQUARES TOTAL (LN)	1.31494
F VALUE	73.72680	DURBIN-WATSON STATISTIC	2.25125
DEGREES OF FREEDOM FOR ERROR	25	DEGREES OF FREEDOM DUE TO REGRESSION	3
TOTAL DEGREES OF FREEDOM	28	NUMBER OF DATA POINTS	29

VARIANCE-COVARIANCE MATRIX

	LN A	B	C	D
LN A	0.23975D-02	-0.38739D-03	-0.27831D-03	-0.23403D-03
B	-0.38739D-03	0.11919D-03	-0.71897D-04	-0.95281D-05
C	-0.27831D-03	-0.71897D-04	0.14497D-03	-0.82843D-04
D	-0.23403D-03	-0.95281D-05	-0.82843D-04	0.22228D-03

Table A-15

COMPANY A: FACTORY OVERHEAD RATE VS EMPLOYMENT AND TIME

a	=	162.59107	b	=	-.00741
c	=	1.67807			
Std. Err. a	=	10.51527	T-Ratio a	=	15.46238
Std. Err. Coeff. b	=	.00232	T-Ratio Coeff. b	=	-3.20069
Std. Err. Coeff. c	=	.70124	T-Ratio Coeff. c	=	2.39301
Beta Coeff. b	=	-.67021	Beta Coeff. c	=	.50109
Std. Err. Est.(Adj)	=	9.29807	Coeff. Var. (Pct)	=	6.27921
Coeff. Det. (Unadj)	=	.57644	Coeff. Corr.(Unadj)	=	.75924
Sum of Sq. y Dev.	=	864.54018	Mean |Pct. y Dev.|	=	4.74230
F Value	=	6.80467	Durbin-Watson d	=	1.56756
Mean of Input y	=	148.07692	Mean of Input w	=	3996.00000
Mean of Input x	=	9.00000			
Std. Dev. Input y	=	13.04199	Std. Dev. Input w	=	1179.34735
Std. Dev. Input x	=	3.89444			
Number of Data Points		13			

Table A-16

COMPANY B: FACTORY OVERHEAD RATE VS EMPLOYMENT AND TIME

a	=	143.26809	b	=	-.00024
c	=	2.08498			
Std. Err. a	=	10.62473	T-Ratio a	=	13.48439
Std. Err. Coeff. b	=	.00085	T-Ratio Coeff. b	=	-.28683
Std. Err. Coeff. c	=	.85512	T-Ratio Coeff. c	=	2.43824
Beta Coeff. b	=	-.07995	Beta Coeff. c	=	.67967
Std. Err. Est.(Adj)	=	9.98208	Coeff. Var. (Pct)	=	6.28859
Coeff. Det. (Unadj)	=	.54621	Coeff. Corr.(Unadj)	=	.73906
Sum of Sq. y Dev.	=	1195.70362	Mean |Pct. y Dev.|	=	5.05478
F Value	=	7.22200	Durbin-Watson d	=	1.65902
Mean of Input y	=	158.73333	Mean of Input w	=	4966.86667
Mean of Input x	=	8.00000			
Std. Dev. Input y	=	13.71895	Std. Dev. Input w	=	4485.41820
Std. Dev. Input x	=	4.47214			
Number of Data Points		15			

Table A-17

COMPANY C: FACTORY OVERHEAD RATE VS EMPLOYMENT AND TIME

a =	132.10156	b =	-.00368
c =	2.09488		
Std. Err. a =	7.81244	T-Ratio a =	16.90912
Std. Err. Coeff. b =	.00158	T-Ratio Coeff. b =	-2.33439
Std. Err. Coeff. c =	.79243	T-Ratio Coeff. c =	2.64362
Beta Coeff. b =	-.53935	Beta Coeff. c =	.61080
Std. Err. Est.(Adj) =	12.44133	Coeff. Var. (Pct) =	9.24319
Coeff. Det. (Unadj) =	.43606	Coeff. Corr.(Unadj)=	.66035
Sum of Sq. y Dev. =	1857.44080	Mean |Pct. y Dev.| =	7.20278
F Value =	4.63948	Durbin-Watson d =	.43227
Mean of Input y =	134.60000	Mean of Input w =	3870.73333
Mean of Input x =	8.00000		
Std. Dev. Input y =	15.33833	Std. Dev. Input w =	2245.45768
Std. Dev. Input x =	4.47214		
Number of Data Points	15		

Table A-18

COMPANY D: FACTORY OVERHEAD RATE VS EMPLOYMENT AND TIME

a =	106.03356	b =	.00205
c =	.41306		
Std. Err. a =	34.60860	T-Ratio a =	3.06379
Std. Err. Coeff. b =	.00151	T-Ratio Coeff. b =	1.35710
Std. Err. Coeff. c =	1.56253	T-Ratio Coeff. c =	.26435
Beta Coeff. b =	.61121	Beta Coeff. c =	.11906
Std. Err. Est.(Adj) =	10.06990	Coeff. Var. (Pct) =	7.26020
Coeff. Det. (Unadj) =	.28519	Coeff. Corr.(Unadj)=	.53403
Sum of Sq. y Dev. =	709.81965	Mean |Pct. y Dev.| =	4.98983
F Value =	1.39641	Durbin-Watson d =	2.35891
Mean of Input y =	138.70000	Mean of Input w =	13812.30000
Mean of Input x =	10.50000		
Std. Dev. Input y =	10.50407	Std. Dev. Input w =	3130.23073
Std. Dev. Input x =	3.02765		
Number of Data Points	10		

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